XVI CONGRESS GAFEVOL 2023

November 28th to December 01th, 2023

TO HONOR THE MEMORY OF **HERNÁN HENRÍQUEZ MIRANDA** (1950 - 2022)

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Welcome

Welcome to Santiago 2023!

We are delighted to welcome friends and colleagues to our home city of Santiago for the XVI GAFEVOL Congress 2023.

Continuing the successful trend of past Congresses, the 2023 Congress in Santiago shows how important the GAFEVOL Congress has become as a regular platform accommodating the rapid pace of progress in the Evolution Equations and Functional Analysis field and for presenting the results of studies which have a direct impact on the applications.

GAFEVOL 2023 will provide a Scientific Programme that builds on the highly successful models from previous Congresses, while incorporating innovative suggestions from valuable stakeholders.

We wish you a pleasant stay in Santiago. Enjoy the Congress!.

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Contents

General Information

Location

The congress will take place in Auditorio Departamento de Matemática y Ciencia de la Computación at the University of Santiago of Chile. The site is indicated with the number 2 in the map.

Useful Phone Numbers

In case of any health emergencies, please call 131 (SAMU)
Police number: 133
Math department of the University of Santiago of Chile: +(56)2 2 718 2033

Lunch

The lunch will take place in Casino de Autoridades–EAO at the University of Santiago of Chile. The site is indicated with the number 7 in the map.

Schedule

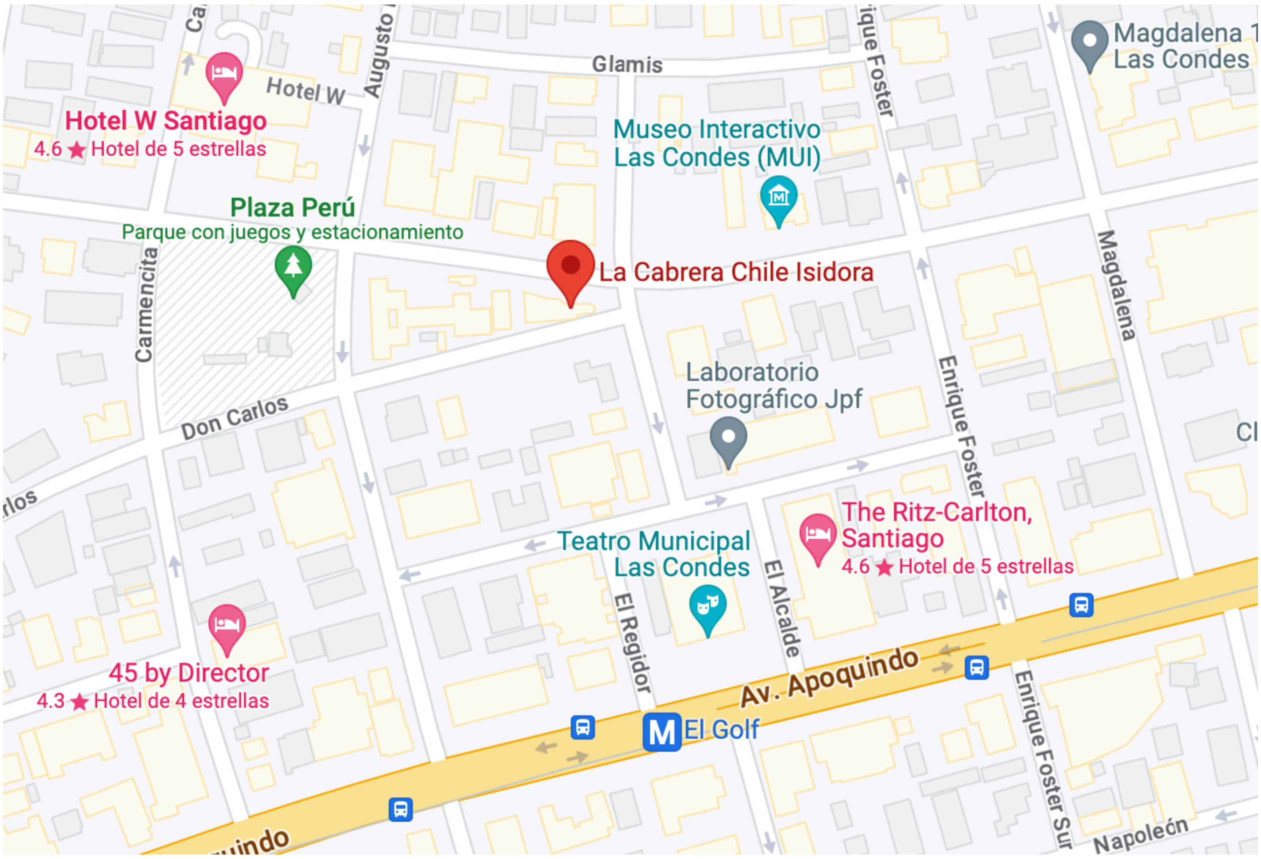
PROGRAM: XVI CONGRESO GAFEVOL: November 28, 29, 30 and December 01, 2023

Auditorio Departamento de Matemática y Ciencia de la Computación

	Tuesday 28	**Wednesday 29**	**Thursday 30**	**Friday 01**
Chairman		*C. Lizama*	*C. Gallegos*	*E. Alvarez*
9:45-10:30	Registration	Jorge Gonzalez	David Urrutia	Eduardo Hernandez
10:30-11:00	Coffee	Coffee break	Coffee break	Coffee break
11:00-11:45	Opening: Vice chancellor; Dean; Associate Dean; Director DMCC	Stiven Diaz	Juan F. Peña	Andrea Prokopczyk
11:45-12:30	Jaqueline Mesquita	Rodrigo Ponce	Gonzalo Robledo	Giovana Siracusa
12:30-14:30	Lunch (Casino Autoridades EAO)	Lunch (Casino Autoridades EAO)	Group photo and lunch (Casino Autoridades EAO)	Lunch (Casino Autoridades EAO)
Chairman/ Chairwoman	*J. Mesquita*	*R. Ponce*	*J. González*	*L. Abadías*
14:30-15:15	Claudio Gallegos	Richar Chacon	Silvia Rueda	Nestor Jara
15:15-16:00	Sergei Trofimchuk	Eduardo Cerpa	Luciano Abadias	Eder Mateus
16:00-16:30	Coffee break	Coffee break	Coffee Break	Coffee Break
16:30-17:15	Carlos Lizama	Humberto Prado	Edgardo Alvarez	Closing ceremony
17:15-18:00				
20:00-24:00				Dinner (La Cabrera Restaurant)

Closing Dinner

The closing dinner will be on December 01th, 2023 at 8:00 PM in the restaurant *La Cabrera*, which is located at Isidora Goyenechea 3275, Las Condes, Santiago.



The principal menu will be the following*

Aperitivo a elegir – *Cocktail to choose from*
Pisco Sour – *Pisco sour*
Espumante – *Sparkling wine*
Cerveza – *Beer*

Entrada – *Starter*
Empanada de carne – *Meat pasty*

Plato principal a elegir – *Main course to choose from*

Punta de Ganso – *Outside round flat*
Bife de Chorizo – *Chorizo steak*
Medio Pollo Grillado – *Grilled chicken*
Ravioles de Espinaca – *Spinach Ravioli*

Ensaladas para compartir – *Side dishes to share*

Ensalada mixta – *Mixed salad*
Papas fritas – *French fries*
Champiñones – *Mushrooms*

Bebestibles – *Drinks*
Dos copas de vino y una bebida incluidos – *Two cups of wine and a soft drink included*

Postre a elegir – *Dessert to choose from*

Flan – *Flan*
Panqueque dulce de leche – *dulce de leche pancake*

Café o infusión – *coffee or infusion*

*Please inform us if you have gluten intolerance, allergy to any condiment or fish or shellfish, if you are diabetic, or if you are vegetarian.

Abstracts of Talks

Spectral sets of generalized Hausdorff matrices on spaces of holomorphic functions on $\mathbb{D}$

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Abstract

Here, we study a family of bounded operators $\mathcal{H}$, acting on Banach spaces of holomorphic functions $X \hookrightarrow \mathcal{O}(\mathbb{D})$, which are subordinated in terms of a C_0-semigroup of weighted composition operators $(v_t C_{\phi_t})$, i.e., $\mathcal{H} = \int_0^\infty v_t C_{\phi_t}\, d\nu(t)$ in the strong sense for some Borel measure ν. This family of operators extends the so-called generalized Hausdorff operators. Here, we obtain the spectrum, point spectrum and essential spectrum of $\mathcal{H}$ under mild assumptions on $(v_t C_{\phi_t}), \nu$ and X. In particular, we obtain these spectral sets for a wide family of generalized Hausdorff operators acting on Hardy spaces, weighted Bergman spaces, weighted Dirichlet spaces and little Korenblum classes. The description for the spectra of the infinitesimal generator of $(v_t C_{\phi_t})$ is the key for our findings. This is a joint work with Jesús Oliva-Maza.

Strongly L^p well-posedness for abstract time-fractional Moore-Gibson-Thompson type equations

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Abstract

We obtain necessary and sufficient conditions for the strongly L^p well-posedness of an abstract evolution equation, arising from fractional Moore-Gibson-Thompson type equations which have recently appeared in the literature. We use Fourier multiplier techniques to derive new characterizations in terms of the R-boundedness of the operator-valued symbol associated to the abstract model, when endowed with the time-fractional Liouville-Grünwald derivative. As a consequence of our characterization, we show novel applications by including several classes of operators other than the Laplacian.

References

[1] E. Alvarez, C. Lizama and M. Murillo-Arcila. *Strongly L^p well-posedness for abstract time-fractional Moore-Gibson-Thompson type equations.* J. Diff. Eq. **376** (2023) 340–369.

Mathematical methods in neurostimulation

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Abstract

Electrical stimulation therapies are used to treat the symptoms of a variety of nervous system disorders. Recently, the use of high frequency signals has received increased attention due to its varied effects on tissues and cells. In this talk, we will see how some mathematical methods can be useful to address relevant questions in this framework when the FitzHugh-Nagumo model of a neuron is considered. Here, the stimulation is through the source term of an ODE and the level of neuron activation is associated with the existence of action potentials which are solutions with a particular profile. A first question concerns the effectiveness of a recent technique called interferential currents, which combines two signals of similar kilohertz frequencies intended to activate deeply positioned cells [1, 2]. The second question is about how to avoid the onset of undesirable action potentials originated when signals that produce conduction block are turned on [3]. We will show theoretical and computational results based on methods such as averaging, Lyapunov analysis, quasi-static steering, and others.

References

[1] E. Cerpa, M. Courdurier, E. Hernández, L. Medina, E. Paduro. *A partially averaged system to model neuron responses to interferential current stimulation*, J. Math. Biology, Vol. 86, No. 1, Article 8, 2023.

[2] E. Cerpa, M. Courdurier, E. Hernández, L. Medina, E. Paduro. *Approximation and stability results for the parabolic FitzHugh-Nagumo system with combined rapidly oscillating sources*, submitted, arXiv:2305.00123v2.

[3] E. Cerpa, N. Corrales, M. Courdurier, L. Medina, E. Paduro. *A*voiding onset activation in neurostimulation with high frequency signals, in preparation.

Tiempos de espera en el movimiento browniano geométrico y distribuciones de probabilidad de tipo ley potencia

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Abstract

William J. Reed en el año 2001 da una justificación convincente de la ley Pareto como modelo para la distribución de la riqueza personal. La riqueza personal la considera como el rendimiento debido a la inversión de un capital financiero con un tiempo de espera aleatorio con distribución exponencial. En lugar de un tiempo de espera exponencial, extendemos el resultado de Reed al considerar tiempos de espera con distribuciones que tienen colas similares a la cola de una variable aleatoria gamma. Las variables aleatorias que resultan de este proceso son de tipo ley potencia más un factor logarítmico. En lugar de ajustar el modelo que emerge de este proceso a la distribución de la riqueza (capital financiero), ajustamos el modelo satisfactoriamente a la producción académica (capital intelectual) de dos conjuntos de datos, uno de matemáticos en universidades mexicanas y otro de matemáticos en universidades estadounidenses.

References

[1] E. P. Balanzario, R. Mendoza, J. Sánchez, The randomly stopped geometric Brownian motion, *Statistics & Probability Letters*, **90** (2014), 85-92.

[2] F. Oberhettinger, Tables of Mellin transforms, *Springer-Verlag*, (1974).

[3] W. J. Reed, The Pareto, Zipf and other power laws, *Economics Letters*, **74** (2001), 15-19.

[4] L. Wasserman, All of statistics. A concise course in statistical inference, *Springer-Verlag*, (2004).

About Appell-type polynomials, their generalizations, and their connection with discrete fractional calculus

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Abstract

In this talk, we will delve into the theory and explore recent developments concerning Appell-type polynomials, as defined in [3]. We will place particular emphasis on well-known classical Appell polynomials such as the Bernoulli polynomials and Euler polynomials, together with their generalizations, as well as their degenerate version, which have been studied over time (see [9, 11]). Additionally, the family of Hermite orthogonal polynomials will be shown as well as some of their properties (see [5]). Drawing inspiration from works [4, 6] and [10], we will present new results pertaining to classes of degenerate unified polynomials. Finally, we will also explore potential connections between these polynomials and discrete fractional calculus [1, 2, 8, 12].

References

[1] E. Alvarez, S. Díaz, C. Lizama, C-Semigroups, subordination principle and the Lvy α-stable distribution on discrete time, *Commun. Contemp. Math.*, **24** (1) (2022).

[2] E. Alvarez, S. Díaz, C. Lizama, Existence of (N, λ)-periodic solutions for abstract fractional difference equations, *Mediterr. J. Math.*, **19** (47) (2022).

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[4] D. Bedoya, C. Cesarano, S. Díaz, W. Ramírez, New classes of degenerate unified polynomials, *Axioms*, **12** (1) (2023).

[5] C. Cesarano, Operational methods and new identities for Hermite polynomials, *Math. Model. Nat. Phenom.* **12** (3) (2017), 44-50.

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[7] C. Cesarano, W. Ramírez, S. Khan, A new class of degenerate Apostol–type Hermite polynomials and applications, *Dolomites Res. Notes Approx*, **15** (1) (2022): 1–10.

[8] C. Lizama, The Poisson distribution, abstract fractional difference equations, and stability, *Proc. Amer. Math. Soc.*, **145** (9) (2017), 3809–3827.

[9] Q.-M. Luo, H. M. Srivastava, *Some generalizations of the ApostolBernoulli and ApostolEuler polynomials*, *J. Math. Anal. Appl.* **308** (1) (2005), 290-302.

[10] W. Ramírez, C. Cesarano, S. Díaz, New results for degenerated generalized Apostol-Bernoulli, Apostol-Euler and Apostol-Genocchi polynomials, *WSEAS Trans. Math.* **21** (2022), 604-608.

[11] H. M. Srivastava, Some formulas for the Bernoulli and Euler polynomials at rational arguments, *Math. Proc. Cambridge Philos. Soc.*, **129** (2000), 7784.

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On a convergence theorem for the Kurzweil-Stieltjes integral

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Abstract

In this talk, we are interested in discussing about one of the fundamental convergence theorems for the Kurzweil–Stieltjes integral [2]. The statement is as follows: Let $g\colon [a,b] \to \mathbb{R}$ be a function of bounded variation. Assume that $f, f_n\colon [a,b] \to \mathbb{R}$ are functions satisfying the following conditions

(a) $\lim_{n\to\infty} f_n(s) = f(s)$ for all $s \in [a,b]$.

(b) For every $n \in \mathbb{N}$, the integral $\int_a^b f_n \mathrm{d}g$ exists in the Kurzweil–Stieltjes sense.

(c) There exists a constant $K > 0$ such that

$$\left| \sum_{j=1}^{l} \int_{\sigma_{j-1}}^{\sigma_j} f_{m_j} \mathrm{d}g \right| \leq K,$$

for every subdivision $\{\sigma_0, \ldots, \sigma_l\}$, and and every finite set $\{m_1, \ldots, m_l\} \subset \mathbb{N}$.

Then the integral $\int_a^b f \mathrm{d}g$ exists and

$$\lim_{n\to\infty} \int_a^b f_n \mathrm{d}g = \int_a^b f \mathrm{d}g.$$

We establish weaker variants of this result to include functions with values in certain Banach spaces. The general case remains an open problem in the field of Kurzweil integration theory.

References

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[2] G. A. Monteiro, A. Slavík, M. Tvrdý, Kurzweil-Stieltjes Integral: Theory and Applications, World Scientific, 2019.

Explicit representation of discrete fractional resolvent families in Banach spaces

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Abstract

In this talk we present the discrete fractional resolvent family $\{S^n_{\alpha,\beta}\}_{n\in\mathbb{N}_0}$ generated by a closed linear operator in a Banach space X for a given $\alpha, \beta > 0$. Moreover, we study its main properties and, as a consequence, we obtain a method to study the existence and uniqueness of the solutions to discrete fractional difference equations in a Banach space.

References

[1] J. González-Camus and R. Ponce, *Explicit representation of discrete fractional resolvent families in Banach spaces.* Fract. Calc. Appl. Anal., Vol. 24, No 6 (2021), pp. 1853–1878.

[2] C. Lizama, *The Poisson distribution, abstract fractional difference equations, and stability,* Proc. Amer. Math. Soc. **145** (2017), no. 9, 3809–3827.

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Explicit Abstract Neutral Differential Equations with State-Dependent Delay: Existence, Uniqueness and Local Well-Posedness

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Abstract

We present some results concerning the existence and qualitative properties of solutions for a class of abstract explicit neutral differential equations with state dependent delay (neutral differential equations in infinite dimensional spaces with delay terms of the form $u'(\sigma(u(t,u(t))))$, $u'(\sigma(u(t,u'(t))))$, $u'(\sigma(u(t,u_t)))$ and so on). Specifically, we note some results on the local and global existence and uniqueness of strict solution and about the local well-posedness for a class of abstract neutral explicit integro-differential equation of the form

$$u'(t) = Au(t) + F\left(t, u(t), \int_0^t K(t,\tau)u'(\sigma(\tau, u'(\tau)))d\tau\right), \quad t \in [0,a], \tag{1}$$

$$u_{|[-p,0]} = \varphi \in \mathcal{B} = C([-p,0];X), \tag{2}$$

where $(X, \|\cdot\|)$ is a Banach space, $A : D(A) \subset X \to X$ is the infinitesimal generator of an analytic semigroup of bounded linear operators $(T(t))_{t\geq 0}$ on X, $K(\cdot)$ is an operator valued map and $F(\cdot)$, $\sigma(\cdot)$ are suitable continuous functions.

The results in this conference are part of our recent paper [1], which extend and generalize the results in the pioneer paper on abstract explicit neutral differential equation with SDD [2]. The other references are about recent results on abstract differential equations with SDD, which were useful to develop the papers [1, 2].

References

[1] E. Hernández, J. Wu, Explicit abstract neutral differential equations with state-dependent delay: Existence, uniqueness and local well-posedness. *J. Differential Equations* 365 (2023), 750-811.

[2] E. Hernández, On explicit abstract neutral differential equations with state-dependent delay. *Proc. Amer. Math. Soc.,* 151 (2023), 1119-1133.

*This work is supported by Fapesp 2021/12559-9.

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[6] E. Hernández, J. Wu, Existence and uniqueness of $\mathbf{C^{1+\alpha}}$-strict solutions for integro-differential equations with state-dependent delay. *Differential and Integral Equations,* 32 (5/6) (2019), 291-322.

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Gohberg lemma and spectral results for pseudodifferential operators on locally compact Abelian groups

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Abstract

We provide a new type of proof for known or new Gohberg lemmas for pseudodifferential operators on Abelian locally compact groups X. We use C^*-algebraic techniques [1], which also give spectral results to which the Gohberg lemma is just a corollary. These results extend most of those appearing in the literature in various directions. In particular, compactness [3] or a Lie [4] structure are not needed. The ideal of all the compact operators in $\mathrm{L}^2(\mathsf{X})$ is replaced by all the ideals having a crossed product structure, which is a consistent generalization. We also indicate several new examples, mostly connected to specific behaviors of functions on the dual Ξ of X.

References

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Brief review of a joint article with H. Henríquez and eventual future directions

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Abstract

Given $a \in L^1(\mathbb{R})$ and the generator A of an L^1-integrable resolvent family of linear bounded operators defined on a Banach space X, we prove the existence of compact almost automorphic solutions of the semilinear integral equation $u(t) = \int_{-\infty}^{t} a(t-s)[Au(s) + f(s, u(s))]ds$ for each $f : \mathbb{R} \times X \to X$ compact almost automorphic in t, for each $x \in X$, and satisfying Lipschitz and Hölder type conditions. In the scalar linear case, we prove that $a \in L^1(\mathbb{R})$ positive, nonincreasing and log-convex is sufficient to obtain existence of compact almost automorphic solutions.

References

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Global existence of solutions for Boussinesq system with energy dissipation

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Abstract

The Boussinesq system coupled by an energy dissipation brings new challenges in the study of global existence of solutions, for instance, this system does not have scale invariance which makes it difficult to show existence of mild solutions when initial data belongs to scaling invariant function spaces. In this paper we are interested to show global existence of solutions $[u,\theta]$ for Boussinesq system coupled by bilinear energy dissipation $\Phi(u) = 2\mu\,\mathcal{E}(u)\cdot\mathcal{E}(u)$ on smooth bounded domain $\Omega \subseteq \mathbb{R}^n$ or whole space $\mathbb{R}^n$, $n \geq 3$, when the initial data $[u_0,\theta_0] \in \mathbf{X}_0 = W^{1,n/2}_{\sigma}(\Omega)\times L^{n/2}(\Omega)$ is sufficiently small and the external force $F(\theta) = \varrho f\theta e_n$ has low regularity in the sense $t^{1/2-b/2n}f \in L^{\infty}\left((0,T) : L^b(\Omega)\right)$ or $f \in L^s((0,T) : L^b(\Omega))$, where $\frac{2}{s} = 1 - \frac{n}{b}$ and $b \in [n,\infty)$. It seems that our results on the global and local well-posedness are the first to provide an L^p-approach when the initial velocity data u_0 belongs to the space $\dot{W}^{1,n/2}_{\sigma}(\mathbb{R}^n)$ that is scaling invariant.

References

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Existence results for abstract functional differential equations with infinite state-dependent delay and applications

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Abstract

In this talk, I will discuss the existence of mild solutions of abstract retarded functional differential equations with infinite state-dependent delay. Also, I will present result concerning the existence of mild solutions for the equations with state-dependent delays as a fixed point of the solution operator of an associated abstract retarded functional differential equation with time-varying delays. I will also apply the results to study the existence of solutions of a state-dependent partial differential equation with infinite state-dependent delay.

References

[1] H. Henríquez, J. G. Mesquita, H. Costa dos Reis, Existence results for abstract functional differential equations with infinite state-dependent delay and applications, *Mathematische Annalen*, to appear, 2023.

A method for the study of h-dichotomies

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Abstract

In this talk we introduce a new method for extending existing results in the theory of exponential dichotomies to dichotomies whose growth is dominated by a differentiable and increasing function h.

This method entails associating a topological group with an invariant measure to each function h. These objects relate to the function h in a way analogous to the relations that exist between the exponential function, the real additive group and the Lebesgue measure, which allows us to rewrite results from the theory of exponential dichotomies into the context of h-dichotomies.

As a showcase of this technique, we give an improvement of existing admissibility results for h-dichotomies.

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Determination of the order in fractional differential equations

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Abstract

The problem of finding or approximating the order in time-fractional differential equations has been widely studied in the last ten years. See for instance [1, 2, 3, 4, 5, 6, 7, 9, 11]. One of the most notable contributions is the paper [3], where authors consider (for $0 < \alpha < 1$) the fractional differential equation for the Caputo fractional derivative

$$\partial_t^\alpha u(x,t) = Au(x,t), \quad x \in \Omega, t > 0 \tag{3}$$

under the initial condition $u(x,0) = u_0(x), x \in \Omega$, where Ω and A are defined as follows: For a bounded open set $\Omega \subset \mathbb{R}^N$ with sufficiently smooth boundary $\partial\Omega$, let X be the Hilbert space $L^2(\Omega)$. On X, the operator $\mathcal{A}$ is defined by $\mathcal{A}u(x) = \sum_{i=1}^N \frac{\partial}{\partial x_i}\left(\sum_{j=1}^N A_{ij}(x)\frac{\partial}{\partial x_i}u(x)\right), u \in X$, where $A_{ij} = A_{ji}$ for any $1 \le i,j \le N$. Suppose that there exists a constant $\gamma > 0$ such that $\sum_{i,j=1}^N A_{ij}(x)\xi_i\xi_j \ge \gamma|\xi|^2$, for all $\xi \in \mathbb{R}^N$ and $x \in \overline{\Omega}$. The operator $A : D(A) \to X$ is defined by

$$(Au)(x) = (\mathcal{A}u)(x), \quad x \in \Omega,$$

where $D(-A) = H^2(\Omega) \cap H_0^1(\Omega)$. The operator $-A$ has a discrete spectrum and its eigenvalues satisfy $0 < \lambda_1 \le \lambda_2 \le \cdots \le \lambda_n \le \cdots$ and $\lim_{n\to\infty} \lambda_n = \infty$. Now, if $\phi_n \in H^2(\Omega) \cap H_0^1(\Omega)$ denotes the normalized eigenfunction associated with $-\lambda_n$, then, by the Fourier method (see [10]), the solution u to (3) is given by

$$u(x,t) = \sum_{n=1}^\infty \langle u_0, \phi_n\rangle_{L^2(\Omega)} E_{\alpha,1}(-\lambda_n t^\alpha)\phi_n(x), \quad x \in \Omega, t > 0, \tag{4}$$

where for any $\alpha, \beta > 0$ and $z \in \mathbb{C}$, $E_{\alpha,\beta}$ denotes the Mittag-Leffler function which is defined by $E_{\alpha,\beta}(z) := \sum_{k=0}^\infty \frac{z^k}{\Gamma(\alpha k+\beta)}$. If $u(t,x)$ denotes the solution to (3), $u_0 \in C_0^\infty(\Omega)$ with $Au_0(x) \neq 0$ and x_0 is a fixed element in Ω, then the order α in (3) is given by (see [3, Theorem 1])

$$\alpha = \lim_{t\to 0^+} \frac{t\frac{\partial u}{\partial t}(x_0,t)}{u(x_0,t) - u_0(x_0)}. \tag{5}$$

A similar result holds for $t \to +\infty$ (see also [3, Theorem 1]). Thus, to determinate the order α we need to know $u(x_0,t)$ and $\frac{\partial u}{\partial t}(x_0,t)$ for $t > 0$ on an interval close to 0 (or $+\infty$). As the authors mention in [8, p. 440], "the problems of the recovery of the fractional orders are far from satisfactory since all the publications either assumed the homogeneous boundary condition or studied this inverse problem by the measurement in $t \in (0,\infty)$." Therefore, the problem of finding the order α in (3) in terms of its solution $u(x,t)$ for a fixed time $t > 0$ remains as an open problem.

In this talk we identify, for small t and a fixed $T > 0$, the order $\alpha > 0$ in the abstract fractional differential equation

$$\partial^\alpha u(t) = Au(t),$$

where the time-fractional derivative ∂^α is understood in the sense of Caputo and Riemann-Liouville, A is a closed (possibly unbounded) linear operator in a Banach space X, and $0 < \alpha < 1$ or $1 < \alpha < 2$.

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The bosonic equation for the wave operator

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Abstract

We study the linear *bosonic equation* $\Box \exp(-c\Box)u = g$, in which $\Box$ is the wave operator on Euclidean space and g is the forcing function. This equation emerges in the study of infinite-derivative linearized gravity and in string theory. We introduce an appropriate class of Hilbert spaces of functions continuously embedded into the scale of Sobolev spaces in which the equation is properly defined. Within this setting, first we give meaning to the operators $exp(-c\Box)$, $c > 0$, where *exp* denotes the exponential function. Then we solve the linear *bosonic equation* explicitly in the strong sense, without using distributions. We also solve this equation via integral transforms, an approach inspired in the theory of the standard wave equation.

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Controllability for differential equations with delay

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Abstract

In this work we study the relationship between the approximate controllability of two classes of delayed differential equations with respect to the approximate controllability of the non delayed equation given by

$$x'(t) = Ax(t) + Bu(t), \quad t \geq 0,$$

where A is the infinitesimal generator of a C_0-semigroup of bounded linear operators in a Banach space X, $x(t) \in X$ for $t \geq 0$, $B : U \to X$ is a bounded linear map and U is a Hilbert space. The first class of equation is described by

$$x'(t) = Ax(t) + L(t)(x_t) + Bu(t), \quad t \geq 0,$$

with $x_t(\theta) = x(t+\theta)$, for $\theta \in [-r, 0]$, and $(L(t))_{t\geq 0}$ being a strongly continuous family of bounded linear operators on X, and the second class is given by the following neutral differential equation

$$\frac{d}{dt}\left(x(t) + F(t)(x_t)\right) = Ax(t) + L(t)(x_t) + Bu(t), \quad t \geq 0,$$

in this case, assuming that A is the infinitesimal generator of an analytic semigroup and $(L(t))_{t\geq 0}$ a strongly continuous family of operator taking values in the domain of a fractional power of A and satisfying a Lipschitz condition on this space.

For both cases we present conditions so that the controllability of the system without delay implies the controllability of the system with delay.

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Sensitivity analysis for time varying ecological networks

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Abstract

We consider an ecological network of N species described by a system of non autonomous ordinary differential equations and determine the total effect of the species j on the species i. To do that, we carry out a sensitivity analysis with respect to press type perturbations and construct a sensitivity matrix $S_{ij}(t)$. Firstly, we will revisit the classical results obtained for the the autonomous case, which have been stated in terms of the equilibria by using the implicit function theorem. Secondly, we provide some useful ways for generalize these sensitivity methods to the non autonomous framework: the main ideas are: i) to work with the recently introduced notion of globally bounded solutions **(GBS)** instead of the classical equilibria, and ii) to assume that the linearization around the **GBS** satisfies the property of exponential dichotomy. Our result extends classical works of theoretical ecology to the nonautonomous case. Finally, we present an example adapted for competitive Lotka-Volterra systems with periodic and almost periodic coefficients.

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Time-step heat problem on the mesh: asymptotic behaviour and decay rates

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Abstract

In this work, we study the asymptotic behaviour and decay of the solution of the fully discrete heat problem. We show basic properties of its solutions, such as the mass conservation principle and their moments, and we compare them to the known ones for the continuous analogue problems. We present the fundamental solution, which is given in terms of spherical harmonics, and we state pointwise and ℓ^p estimates for that. Such considerations allow to prove decay and large time behaviour results for the solutions of the fully discrete heat problem, giving the corresponding rates of convergence on ℓ^p spaces.

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On evolutionary Volterra equations with state-dependent delay

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Abstract

In this work we study some topological properties of the solution set for a class of integro-differential equations with state-dependent delay

$$\begin{cases} u'(t) &= \int_0^t a(t-s)Au(s)ds + f\left(t, u_{\rho(t,u_t)}\right), \quad t \in [0,b], \\ u(0) &= \varphi \in \mathfrak{B}, \end{cases} \tag{6}$$

where $A : D(A) \subset X \to X$ is a closed linear operator defined on a Banach space X, the kernel $a \in L^1_{loc}((0,\infty))$ and the history $u_t : (-\infty, 0] \to X$, given by

$$u_t(\theta) = u(t+\theta),$$

belongs to some abstract phase space $\mathfrak{B}$ described axiomatically. Furthermore, $f : [0,b] \times \mathfrak{B} \to X$ and $\rho : [0,b] \times \mathfrak{B} \to (-\infty, b]$ are given functions. From the mathematical point of view, we are motivated by elegance and simplicity that evolutionary integro-differential equations of the type (6) provides to problems in mathematical physics.

As typical application of (6) we consider the problem

$$\begin{cases} u_t(t,x) = \int_0^t da(s)u_{xx}(t-s,x) + h\big(t, x, u(t-\sigma(\|u(t,x_0)\|), x)\big), \ t \geq 0, \ x \in [0,\pi], \\ u(t,0) = u(t,\pi) = 0, \quad t > 0, \\ u(t,x) = \varphi(t,x), \quad t \leq 0, \quad x \in [0,\pi], \end{cases}$$

where $x_0 \in (0,\pi)$ is fixed, $a : [0,\infty) \to (0,\infty)$ is a function of bounded variation on each compact interval $J = [0,T]$, $T > 0$, with $a(0) = 0$, and

$$\sigma : [0,\infty) \to [0,\infty)$$

is a continuous function. This type of equations has been the subject of many research papers in the last years since it has applications in such different fields as the theory of viscoelastic materials, thermodynamics, electrodynamics and population biology, cf. [1, 2, 3, 4, 5, 6, 7, 8] and references therein.

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Dynamics of one-dimensional maps and Gurtin-MacCamy's population model

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Abstract

Motivated by Ma and Magal recent work [1] on the global stability property of the Gurtin-MacCamy's population model, we consider a family of scalar nonlinear convolution equations with unimodal nonlinearities. Particularly we relate Ivanov and Sharkovsky analysis of singularly perturbed delay differential equations in [2] with the asymptotic behaviour of solutions of the Gurtin-MacCamy's system. By the classification proposed in [2], we can distinguish three fundamental kinds of continuous solutions of our equations: asymptotically constant type, relaxation type and turbulent type solutions. In this talk (based on our recent studies [3]) we present various conditions assuring that all solutions belong to the first of the mentioned three classes. In the setting of unimodal convolution equations, these conditions suggest a generalised version of the famous Wright's conjecture [4].

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Some results about the Lin's Homeomorphism

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Abstract

Let us consider two systems of ordinary differential equations. Firstly, the nonautonomous and non-linear system

$$\dot{y} = C(t)y + B(t)y + g(t,y) \quad t \in \mathbb{R}, \tag{7}$$

and the autonomous linear system

$$\dot{z} = -\frac{\delta}{2} I z, \tag{8}$$

where $\delta > 0$ and $I \in M_n(\mathbb{R})$ is the identity matrix. In addition $C, B : \mathbb{R} \to M_n(\mathbb{R})$ and $g : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$ are continuous and verify the following conditions:

(A1) $t \mapsto C(t)$ is bounded on $\mathbb{R}$ and for any $t \in \mathbb{R}$, we have $C(t) :=\text{diag}\{C_1(t), C_2(t), \ldots, C_n(t)\}$ such that,

$$C_i(t) \leq -\delta \quad \text{for all } t \in \mathbb{R} \text{ and any } i \in \{1,\ldots,n\} \quad \text{and} \quad ||B(t)|| \leq \frac{\delta}{4}.$$

(A2) The function g verifies $g(t,0) = 0$ for any $t \in \mathbb{R}$ and,

$$|g(t,y_1) - g(t,y_2)| \leq \frac{\delta}{4}|y_1 - y_2| \quad \text{for all } t \in \mathbb{R} \text{ and any } y_1, y_2 \in \mathbb{R}^n,$$

where $|\cdot|$ is the euclidean norm in $\mathbb{R}^n$ while $||\cdot||$ is the induced operator norm by $|\cdot|$.

By considering the above conditions, Faxing Lin (2007) proved in [1] that the nonlinear system (7) is *topologically equivalent* [1, 2] to the linear system (8), namely, there exists a map $H : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$ satisfying the following properties:

(i) $\lim_{y \to y_0} H(t,y) = H(t,y_0)$ and $\lim_{||y|| \to \infty} ||H(t,y)||^{-1} = 0$, uniforly with respect to t,

(ii) For any fixed $t \in \mathbb{R}$, $H_t : \mathbb{R}^n \to \mathbb{R}^n$ defined by $H_t(y) = H(t,y)$ is an homeomorphism,

(iii) The function $G : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$ given by $G(t,z) = H_t^{-1}(z)$ verifies (i),

(iv) $t \mapsto H(t, y(t))$ is a solution of (8) when $t \mapsto y(t)$ is a solution of (7),

(v) $t \mapsto G(t, z(t))$ is a solution of (7) when $t \mapsto z(t)$ is solution of (8).

The topological equivalence function is given by the function $H : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$ defined by:

$$H(\tau,y) = \begin{cases} Y(T(\tau,y),\tau,y)\exp\left(\frac{\delta}{2}(T(\tau,y)-\tau)\right) & \text{for } y \neq 0 \\ 0 & \text{for } y = 0, \end{cases} \tag{9}$$

where $t \mapsto Y(t,\tau,y)$ is the solution of (7) with initial condition y at time $t = \tau$ and $T(\tau,y)$ verifies

$$||Y(T(\tau,y),\tau,y)||^2 = 1 \quad \text{for all } y \in \mathbb{R} \setminus \{0\} \text{ y } \tau \in \mathbb{R},$$

and the function $T : \mathbb{R} \times (\mathbb{R}^n \setminus \{0\}) \to \mathbb{R}$ is called by *crossing time function*.

The topological equivalence property defines a homeomorphism family $H_\tau : \mathbb{R}^n \to \mathbb{R}^n$ defined by $H_\tau(y) = H(\tau, y)$ for all $\tau \in \mathbb{R}$ and $y \in \mathbb{R}$, which is called *Lin's Homeomorphism.*

Curiously, the smoothness properties of Lin's homeomorphism have not been studied in the literature. In this context, our goal to find a set of necessary conditions ensuring that $H, G \in C^k(\mathbb{R}\times(\mathbb{R}^n\setminus\{0\}), \mathbb{R}^n)$ with $k \geq 1$, that is, each Lin's homeomorphism H_τ be a preserving orientation diffeomorphism of class $C^k(\mathbb{R}^n \setminus \{0\}, \mathbb{R}^n \setminus \{0\})$. In this way, sufficient conditions are studied to determine the smoothness properties of the crossover time function. To achieve this goal, tools such as the smoothness properties of the solution of an ODE with respect to the initial conditions, the implicit function theorem and the theory of uniform asymptotic stability will be required.

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